1-AM Concepts

Using trigonometric functions, we can express the sine wave carrier with the simple expression

$$\upsilon c = Vc \sin 2\pi fct$$

In this expression, υc represents the instantaneous value of the carrier sine wave voltage at some specific time in the cycle; Vc represents the peak value of the constant un-modulated carrier sine wave as measured between zero and the maximum amplitude of either the positive-going or the negative-going alternations (Fig. 1); fc is the frequency of the carrier sine wave; and t is a particular point in time during the carrier cycle.

A sine wave modulating signal can be expressed with a similar formula

$$\upsilon m = Vm \sin 2\pi fmt$$

where

υm = instantaneous value of information signal

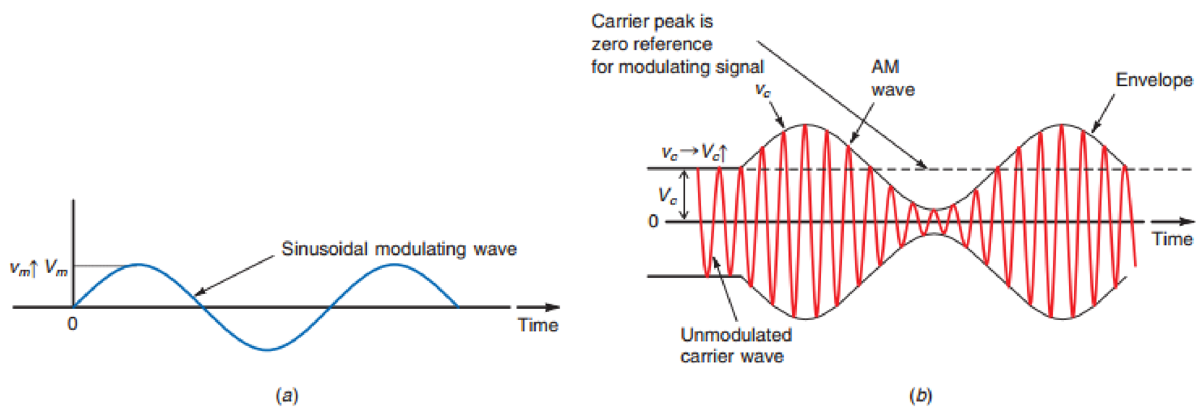
Vm = peak amplitude of information signal

fm = frequency of modulating signal

In amplitude modulation, it is particularly important that the peak value of the modulating signal be less than the peak value of the carrier. Mathematically,

$$Vm < Vc$$

Values for the carrier signal and the modulating signal can be used in a formula to express the complete modulated wave.



Amplitude Modulation Fundamentals

Fig.1 Amplitude modulation. (a) The modulating or information signal. (b) The modulated carrier

First, keep in mind that the peak value of the carrier is the reference point for the modulating signal; the value of the modulating signal is added to or subtracted from the peak value of the carrier. The

instantaneous value of either the top or the bottom voltage envelope v_1 can be computed by using the equation

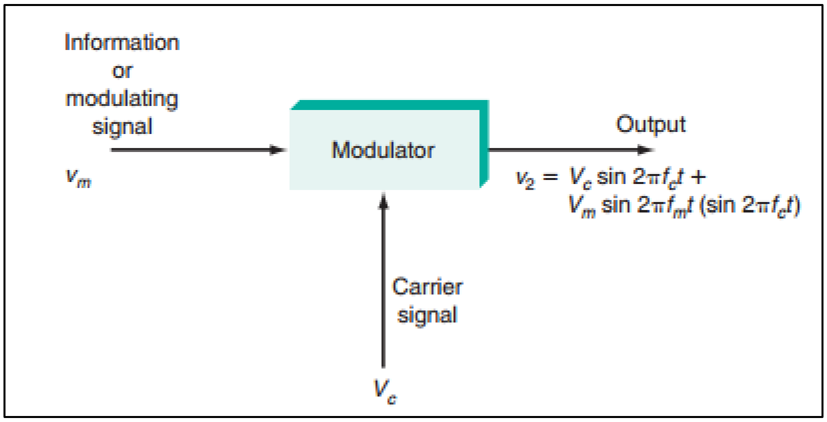
$$v1 = V_c + v_m = V_c + V_m \sin 2\pi f_m t$$

which expresses the fact that the instantaneous value of the modulating signal algebraically adds to the peak value of the carrier. Thus, we can write the instantaneous value of the complete modulated wave $v2$ by substituting $v1$ for the peak value of carrier voltage V_c as follows:

$$v2 = v1 \sin 2\pi f_c t$$

Now substituting the previously derived expression for V_1 and expanding, we get the following:

$\upsilon 2 = (Vc + Vm \sin 2\pi fmt) \sin 2\pi fct = Vc \sin 2\pi fct + (Vm \sin 2\pi fmt)(\sin 2\pi fct)$



Amplitude modulator showing input and output signals.

where $\upsilon 2$ is the instantaneous value of the AM wave (or υ_{AM}), $Vc \sin 2\pi fct$ is the carrier waveform, and $(Vm \sin 2\pi fmt)(\sin 2\pi fct)$ is the carrier waveform multiplied by the modulating signal waveform.

2- Modulation Index and Percentage of Modulation

As stated previously, for undistorted AM to occur, the modulating signal voltage *Vm* must be less than the carrier voltage *Vc*. Therefore, the relationship between the amplitude of the modulating signal and the amplitude of the carrier signal is important. This relationship, known as the modulation index m (also called the modulating factor or coefficient, or the degree of modulation), is the ratio

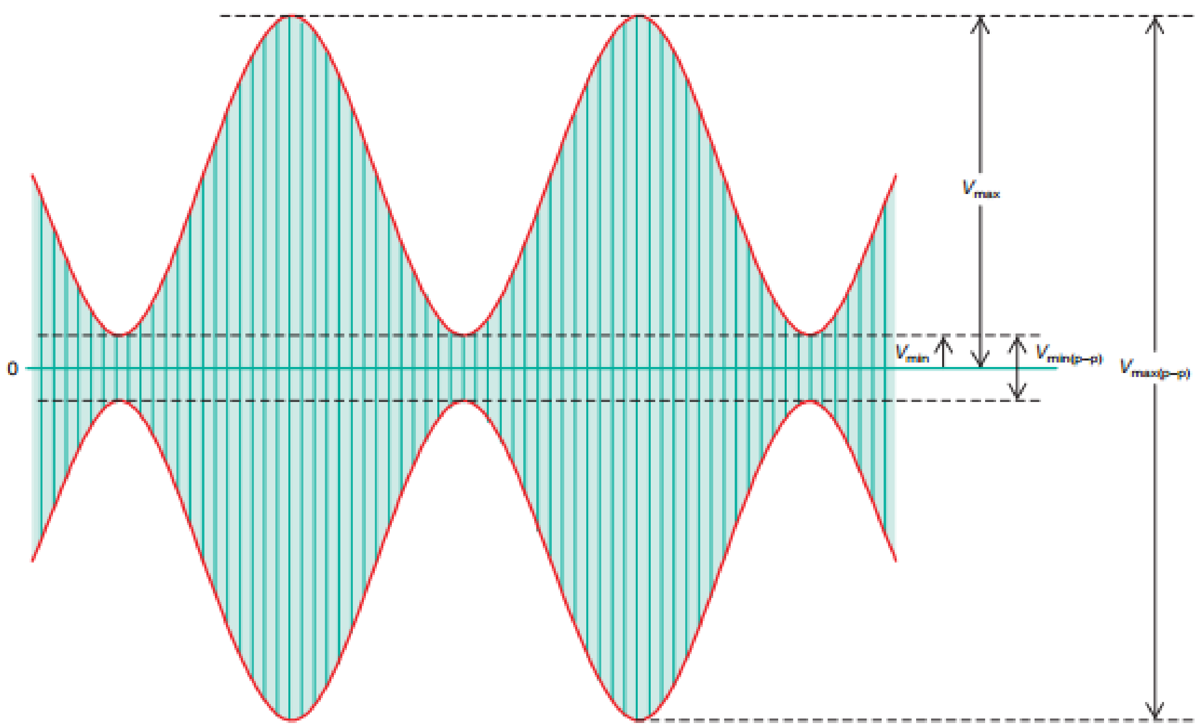
$$m = \frac{V_m}{V_c}$$

Multiplying the modulation index by 100 gives the percentage of modulation.

For example, if the carrier voltage is 9 V and the modulating signal voltage is 7.5 V, the modulation factor is 0.8333 and the percentage of modulation is 0.833 × 100 = 83.33%.

3- Percentage of Modulation

The modulation index can be determined by measuring the actual values of the modulation voltage and the carrier voltage and computing the ratio. However, it is more common to compute the modulation index from measurements taken on the composite modulated wave itself. When the AM signal is displayed on an oscilloscope, the modulation index can be computed from Vmax and Vmin, as shown in Fig. 3-5. The peak value of the modulating signal Vm is one-half the difference of the peak and trough values:



$$V_m = \frac{V_{max} - V_{min}}{2}$$

As shown in Fig., Vmax is the peak value of the signal during modulation, and Vmin is the lowest value, or trough, of the modulated wave. The Vmax is one-half the peak-to-peak value of the AM signal, or Vmax(p-p)/2. Subtracting Vmin from Vmax produces the peakto-peak value of the modulating signal. One-half of that, of course, is simply the peak value.

The peak value of the carrier signal Vc is the average of the Vmax and Vmin values:

$$V_c = \frac{V_{max} + V_{min}}{2}$$

The modulation index is

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

Example

Suppose that on an AM signal, the Vmax (p-p) value read from the graticule on the oscilloscope screen is 5.9 divisions and Vmin(p–p) is 1.2 divisions, find

a. What is the modulation index?

b. Calculate Vc, Vm, and m if the vertical scale is 2 V per division?

a. What is the modulation index?

$$m \frac{V_{max} - V_{min}}{V_{max} + V_{min}} = \frac{5.9 - 1.2}{5.9 + 1.2} = \frac{4.7}{7.1} = 0.662$$

b. Calculate Vc, Vm, and m if the vertical scale is 2 V per division?

$$V_c = \frac{V_{max} + V_{min}}{2} = \frac{5.9 + 1.2}{2} = \frac{7.1}{2} = 3.55 \text{ @ } \frac{2\text{ V}}{\text{div}}$$

$$V_c = 3.55 \times 2\text{ V} = 7.1\text{ V}$$

$$V_m = \frac{V_{max} - V_{min}}{2} = \frac{5.9 - 1.2}{2} = \frac{4.7}{2}$$

$$= 2.35 \text{ @ } \frac{2\text{ V}}{\text{div}}$$

$$V_m = 2.35 \times 2\text{ V} = 4.7\text{ V}$$

$$m = \frac{V_m}{V_c} = \frac{4.7}{7.1} = 0.662$$

4- Sidebands and the Frequency Domain

Whenever a carrier is modulated by an information signal, new signals at different frequencies are generated as part of the process. These new frequencies, which are called side frequencies, or sidebands, occur in the frequency spectrum directly above and directly below the carrier frequency. More specifically, the sidebands occur at frequencies that are the sum and difference of the carrier and modulating frequencies. When signals of more than one frequency make up a waveform, it is often better to show the AM signal in the frequency domain rather than in the time domain.

Sideband Calculations

When only a single-frequency sine wave modulating signal is used, the modulation process generates two sidebands. If the modulating signal is a complex wave, such as voice or video, a whole range of frequencies modulate the carrier, and thus a whole range of sidebands are generated.

The upper sideband f_{USB} and lower sideband f_{LSB} are computed as

$f_{USB} = f_c + f_m$ and $f_{LSB} = f_c - f_m$

where fc is the carrier frequency and fm is the modulating frequency.

The existence of sidebands can be demonstrated mathematically, starting with the equation for an AM signal described previously:

$v_{AM} = V_c \sin 2\pi f_c t + (V_m \sin 2\pi f_m t)(\sin 2\pi f_c t)$

By using the trigonometric identity that says that the product of two sine waves is

$$\sin A \sin B = \frac{\cos(A - B)}{2} - \frac{\cos(A + B)}{2}$$

and substituting this identity into the expression a modulated wave, the instantaneous amplitude of the signal becomes

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2}\cos 2\pi t(f_c - f_m) - \frac{V_m}{2}\cos 2\pi t(f_c + f_m)$$

where the first term is the carrier; the second term, containing the difference fc - fm, is the lower sideband; and the third term, containing the sum fc + fm, is the upper sideband.

For example, assume that a 400-Hz tone modulates a 300-kHz carrier. The upper and lower sidebands are

$f_{USB} = 300{,}000 + 400 = 300{,}400$ *Hz or 300.4 kHz*

$f_{LSB} = 300{,}000 - 400 = 299{,}600$ *Hz or 299.6 kHz*

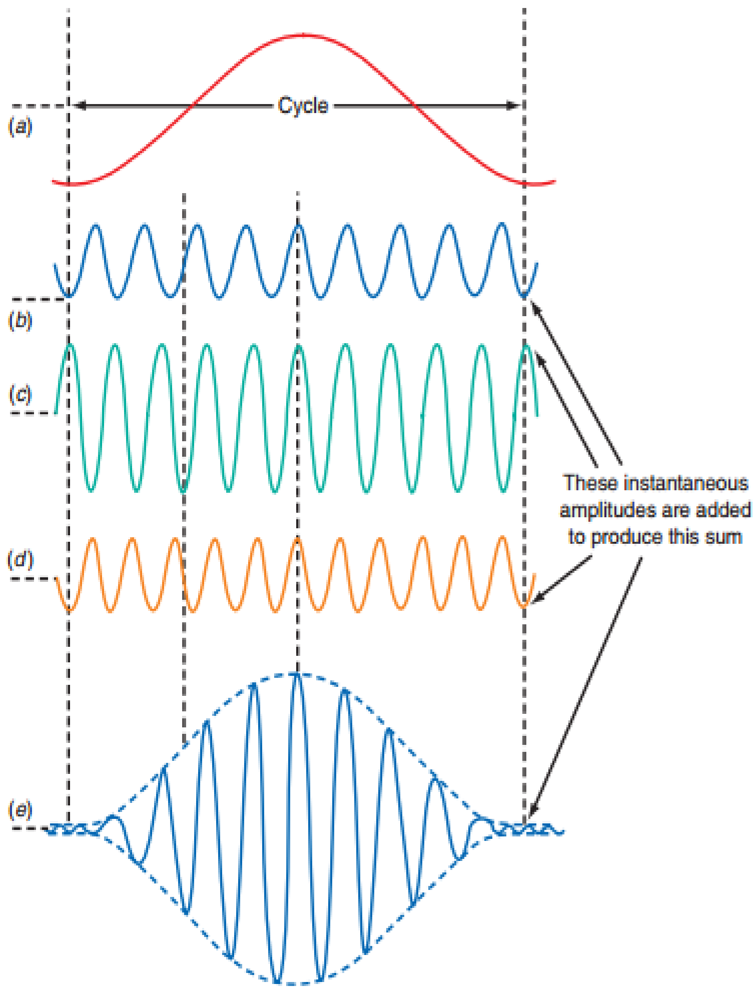


Fig. The AM wave is the algebraic sum of the carrier and upper and lower sideband sine waves. (a) Intelligence or modulating signal. (b) Lower sideband. (c) Carrier. (d) Upper sideband. (e) Composite AM wave.

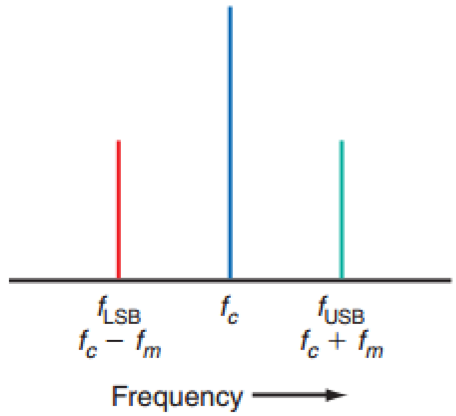


Figure: A frequency-domain display of an AM signal (voltage

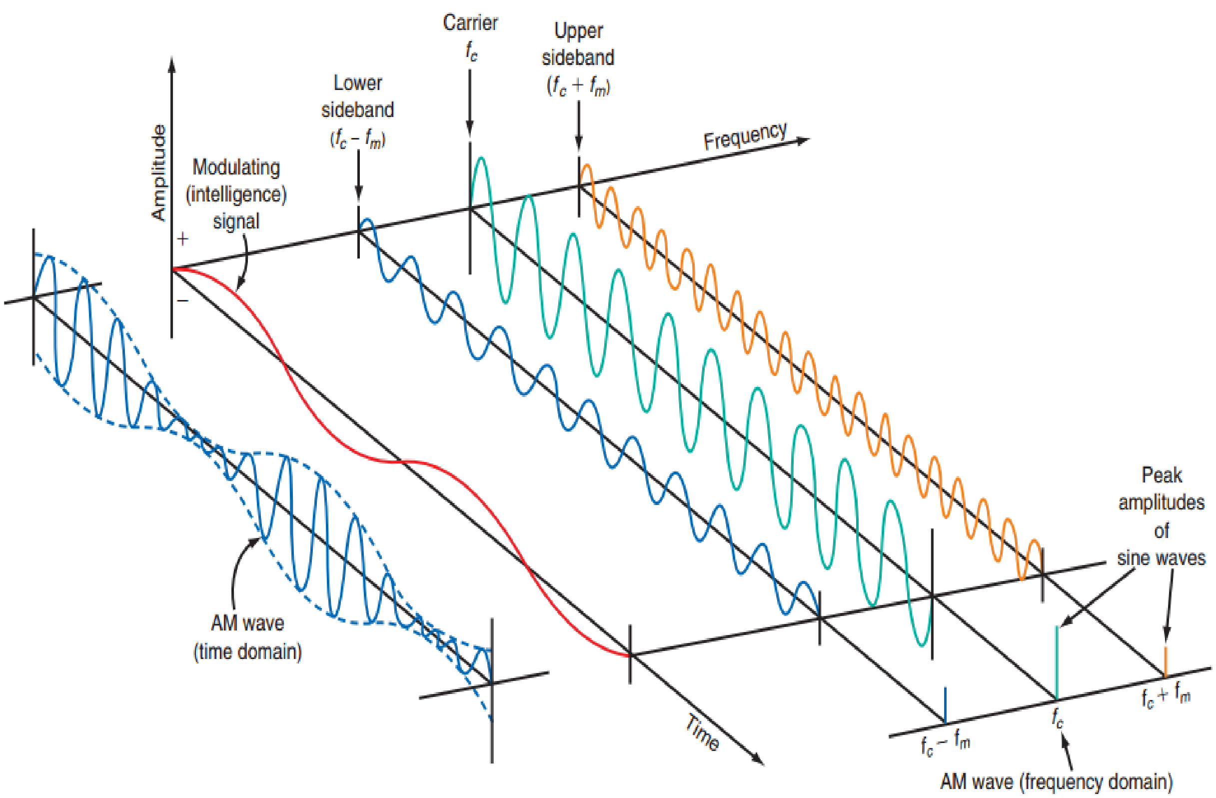


Figure: The relationship between the time and frequency domains.

The total bandwidth is simply the difference between the upper and lower sideband frequencies:

$$BW = f_{USB} - f_{LSB}$$

$$BW = 2f_m.$$

Example

A standard AM broadcast station is allowed to transmit modulating frequencies up to 5 kHz. If the AM station is transmitting on a frequency of 980 kHz, compute the maximum and minimum upper and lower sidebands and the total bandwidth occupied by the AM station.

Sol:

$$f_{USB} = 980 + 5 = 985 \text{ kHz}$$

$$f_{LSB} = 980 - 5 = 975 \text{ kHz}$$

$$BW = f_{USB} - f_{LSB} = 985 - 975 = 10 \text{ kHz} \quad \text{or}$$

$$BW = 2(5 \text{ kHz}) = 10 \text{ kHz}$$

5- AM Power

In radio transmission, the AM signal is ampliied by a power amplii er and fed to the antenna with a characteristic impedance that is ideally, but not necessarily, almost pure resistance. The AM signal is really a composite of several signal voltages, namely, the carrier and the two sidebands, and each of these signals produces power in the antenna. The total transmitted power PT is simply the sum of the carrier power Pc and the power in the two sidebands PUSB and PLSB:

$$P_T = P_c + P_{LSB} + P_{USB}$$

You can see how the power in an AM signal is distributed and calculated by going back to the original AM equation:

$$v_{AM} = V_c \sin 2\pi f_c t + \frac{V_m}{2}\cos 2\pi t(f_c - f_m) - \frac{V_m}{2}\cos 2\pi t(f_c + f_m)$$

Finally, we get a handy formula for computing the total power in an AM signal when the carrier power and the percentage of modulation are known:

$$P_T = P_c\left(1 + \frac{m^2}{2}\right)$$

Example

An AM transmitter has a carrier power of 30 W. The percentage of modulation is 85 percent. Calculate (a) the total power and (b) the power in one sideband.

a. $P_T = P_c\left(1 + \frac{m^2}{2}\right) = 30\left[1 + \frac{(0.85)^2}{2}\right] = 30\left(1 + \frac{0.7225}{2}\right)$

$P_T = 30(1.36125) = 40.8$ W

b. P_{SB} (both) $= P_T - P_c = 40.8 - 30 = 10.8$ W

P_{SB} (one) $= \frac{P_{SB}}{2} = \frac{10.8}{2} = 5.4$ W