

Thermal Noise Power:

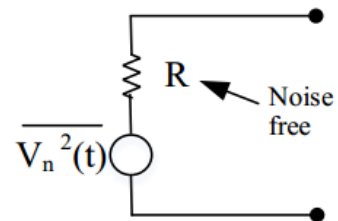
The thermal noise power in a resistor R can be found either using voltage model or current model.

Voltage model:

$$\overline{V_n^2(t)} = \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} 2kTRd\omega \quad = 4kTRB \text{ volt}^2$$

(Or $4kTB$ watt)

$$\text{r.m.s noise voltage} = \sqrt{\overline{V_n^2(t)}} \text{ volt}$$

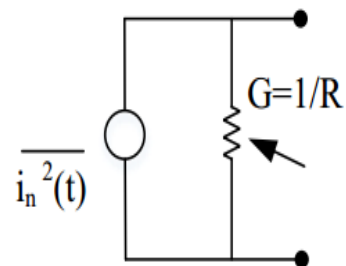


Current model:

$$\overline{I_n^2(t)} = \frac{1}{2\pi} \int_{-2\pi B}^{2\pi B} 2kTRd\omega \quad = 4kTRB \text{ volt}^2$$

(Or $4kTB$ watt)

$$\text{r.m.s noise voltage} = \sqrt{\overline{I_n^2(t)}} \text{ volt}$$



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Ex 1:

Calculate the r.m.s noise voltage arising from thermal noise in two resistors 100 Ω and 150 Ω at T=300 ok:

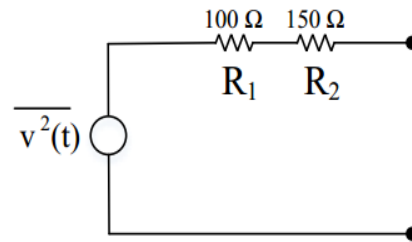
a) Connected in series

b) Connected in parallel

Solution:

$$\begin{aligned} \text{a) } \overline{v_n^2(t)} &= 4kTRB \text{ v}^2 \\ &= 4kTB(R_1 + R_2) = 4 \times 1.38 \times 10^{-23} \times 10^6 \times (100 + 150) \times 300 \\ &= 4.14 \times 10^{-12} \text{ volt}^2 \end{aligned}$$

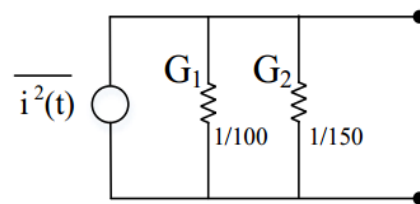
$$V_{n \text{ rms}} = \sqrt{\overline{v_n^2(t)}} = 2.3 \text{ } \mu\text{v}$$



$$\begin{aligned} \text{b) } \overline{i_n^2(t)} &= 4kTGB \text{ amp}^2 \\ &= 4kTB(G_1 + G_2) \\ &= 4 \times 1.38 \times 10^{-23} \times 300 \times 10^6 \times \left(\frac{1}{100} + \frac{1}{150}\right) \end{aligned}$$

$$= 2.76 \times 10^{-16} \text{ ampers}^2$$

$$i_{n \text{ rms}} = \sqrt{\overline{i_n^2(t)}} = 0.0166 \text{ } \mu\text{A}$$



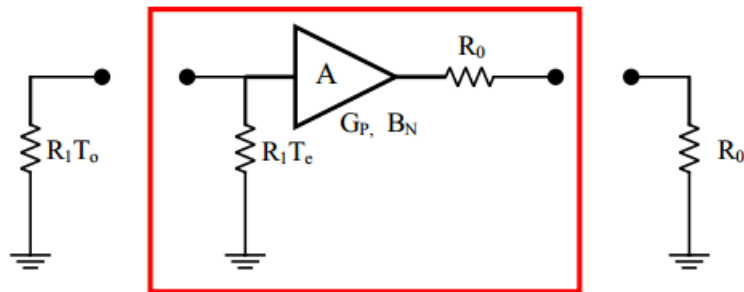
The equivalent parallel resistance $R_{eq} = \frac{1}{G_1 + G_2} = 60 \text{ } \Omega$

$$\begin{aligned} \therefore V_{rms} &= i_{n \text{ rms}} \times R_{eq} \\ &= (0.166 \times 10^{-6}) \times 60 \\ &= 0.997 \text{ } \mu\text{V} \end{aligned}$$

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Thermal noise in Amplifiers:

The thermal noise power in amplifier having bandwidth B and Gain G_p is referred to what is so called “noise temperature T_e ” raised at amplifier input resistance



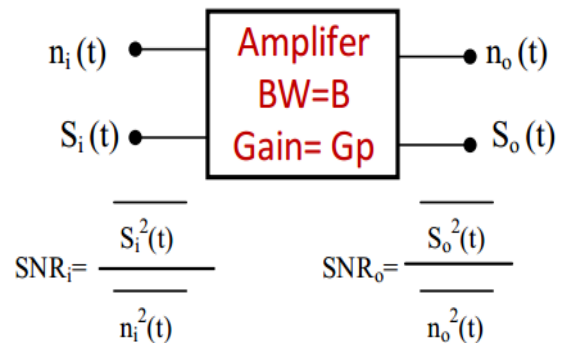
Available noise power in amplifier

$$P_a = KT_e B ..$$

Noise Figure:

It is a ratio of signal-to-noise ratio at amplifier input to the signal-to-noise ratio at amplifier output.

$$F = \frac{SNR_i}{SNR_o} \geq 1$$



For ideal amplifier

$$F=1$$

Input noise power: $N_i = kT_o B$ (referred to resistance connected to amplifier at ..

Input signal power: S_i

...room temperature $0^{12} c^{-0}25 c)$

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Output noise power:

$$N_o = kT_oBG_P + kT_eBG_P$$

Output signal power:

$$S_o = S_iG_P$$

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{S_i/kT_oB}{S_iG_P/(kT_oBG_P + kT_eBG_P)}$$

$$F = 1 + \frac{T_e}{T_o}$$

Notes:

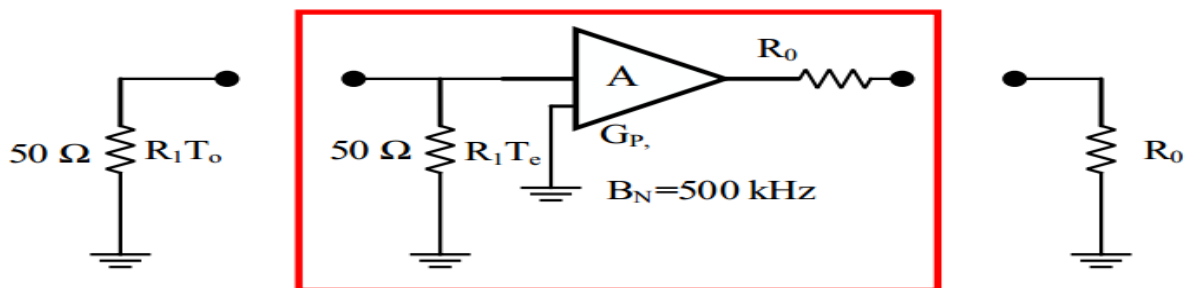
1- Sometimes F is represented in decibles

$$F_{dB} = 10 \log_{10} F \quad \dots$$

$$F = 10^{\left(\frac{F_{dB}}{10}\right)} \quad \dots$$

Ex2:

A given amplifier has 4dB noise figure, a noise bandwidth of 500 kHz and an input resistance of 50 Ω. Calculate the rms signal input, which yields an output signal-to-thermal noise ratio of unity when the amplifier is connected to a 50-Ω input at 290k.



Solution:

$$N_i = kT_o B = 1.38 \times 10^{-23} \times 290 \times 500 \times 10^3 = 2 \times 10^{-15} \text{ watt}$$

$$\overline{n_i^2(t)} = N_i \times R = 50 \times 2 \times 10^{-15} = 1 \times 10^{-13} \text{ volt}^2$$

$$F=4 \text{ dB} = 2.51$$

$$F = \frac{(S/N)_i}{(S/N)_o} \Rightarrow (S/N)_i = F(S/N)_o = F \times 1 = F$$

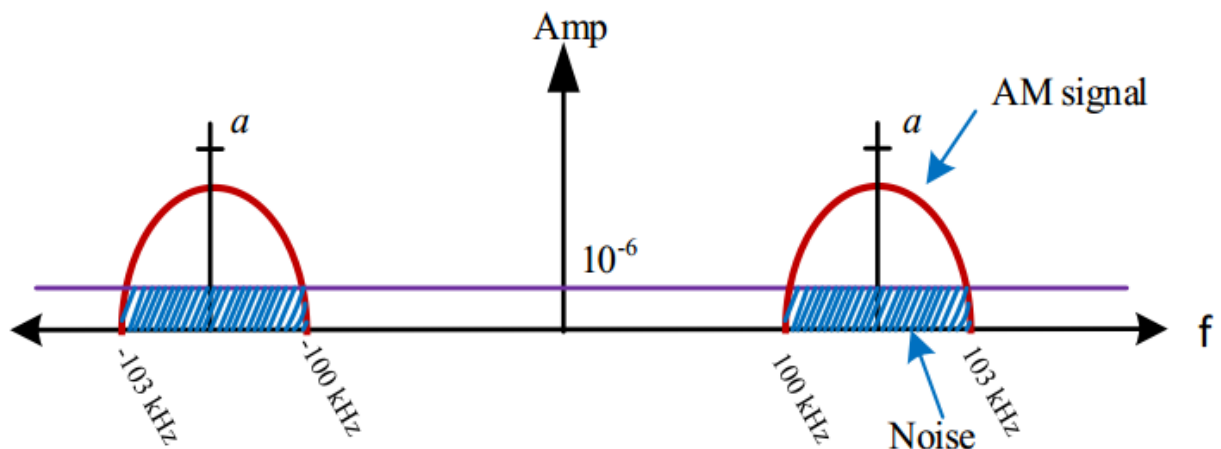
$$\frac{\overline{S_i^2(t)}}{\overline{n_i^2(t)}} = 2.51 \Rightarrow \overline{S_i^2(t)} = 2.51 \times 10^{-13} \text{ volt}^2$$

$$\therefore V_{rms_{si}} = \sqrt{\overline{S_i^2(t)}} = 0.501 \mu v$$

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Ex 3:

An AM signal of 50 watt power is transmitted in a frequency range 100-103 kHz in a transmission channel. If the additive noise power spectral density (two sided) in a transmission channel is 1μ watt/Hz. Find the signal-to-noise ratio in the transmission channel.



Solution:

$$N = \frac{2}{2\pi} \int_{200\pi \cdot 10^3}^{206\pi \cdot 10^3} S_n(\omega) d\omega = \frac{1}{\pi} \int_{200\pi \cdot 10^3}^{206\pi \cdot 10^3} 10^{-6} d\omega = 6 \times 10^{-3} \text{ watt}$$

$$SNR = \frac{S}{N} = \frac{50}{6 \times 10^{-3}} = 8333.33$$

$$SNR_{dB} = 10 \log 8333.33 = 39.208 \text{ dB}$$