

2.7 Binary Arithmetic

Binary arithmetic is essential in all types of digital systems. To understand these systems, you must know the basics of binary addition, subtraction, multiplication, and division.

2.7.1 Binary Addition

The four basic rules for adding binary digits (bits) are as follows:

$$\begin{aligned} 0 + 0 &= 0 \text{ Sum of 0 with a carry of 0} \\ 0 + 1 &= 1 \text{ Sum of 1 with a carry of 0} \\ 1 + 0 &= 1 \text{ Sum of 1 with a carry of 0} \\ 1 + 1 &= 10 \text{ Sum of 0 with a carry of 1} \end{aligned}$$

Notice that the first three rules result in a single bit and in the fourth rule the addition of two 1s yields a binary two (10). When binary numbers are added, the last condition creates a sum of 0 in a given column and a carry of 1 over to the next column to the left, as illustrated in the following examples:

Example: Add $11 + 1$

Sol.

$$\begin{array}{r} \text{Carry Carry} \\ \begin{array}{r} 1 \leftarrow \\ 0 \quad 1 \quad 1 \\ + 0 \quad 0 \quad 1 \\ \hline 1 \quad 0 \quad 0 \end{array} \end{array}$$

In the right column, $1 + 1 = 0$ with a carry of 1 to the next column to the left. In the middle column, $1 + 1 + 0 = 0$ with a carry of 1 to the next column to the left. In the left column, $1 + 0 + 0 = 1$.

Carry bits \rightarrow

$$\begin{aligned} 1 + 0 + 0 &= 0 \quad 1 \text{ Sum of 1 with a carry of 0} \\ 1 + 1 + 0 &= 1 \quad 0 \text{ Sum of 0 with a carry of 1} \\ 1 + 0 + 1 &= 1 \quad 0 \text{ Sum of 0 with a carry of 1} \\ 1 + 1 + 1 &= 1 \quad 1 \text{ Sum of 1 with a carry of 1} \end{aligned}$$

Example: Add $111 + 11$

Sol.

$$\begin{array}{r}
 \text{Carry} \quad \text{Carry} \\
 \begin{array}{cccc}
 & & 1 & 1 \\
 & 1 & 1 & 1 \\
 + & & 1 & 1 \\
 \hline
 1 & 0 & 1 & 0
 \end{array}
 \end{array}$$

2.7.2 Binary Subtraction

The four basic rules for subtracting bits are as follows:

$$0 - 0 = 0$$

$$1 - 1 = 0$$

$$1 - 0 = 1$$

$$10 - 1 = 1 \quad 0 - 1 \text{ with a borrow of } 1$$

When subtracting numbers, you sometimes have to borrow from the next column to the left. **A borrow is required in binary only when you try to subtract a 1 from a 0.** In this case, when a 1 is borrowed from the next column to the left, a 10 is created in the column being subtracted, and the last of the four basic rules just listed must be applied.

Example: Subtract 011_2 from 101_2 .

Sol.

Left column:

When a 1 is borrowed,
a 0 is left, so $0 - 0 = 0$.

Middle column:

Borrow 1 from next column
to the left, making a 10 in
this column, then $10 - 1 = 1$.

$$\begin{array}{r}
 \begin{array}{ccc}
 0 & 1 & 0 \\
 \swarrow & \boxed{10} & \leftarrow 1 \\
 1 & 1 & 0
 \end{array} \\
 - \begin{array}{ccc}
 0 & 1 & 1 \\
 \hline
 0 & 1 & 0
 \end{array}
 \end{array}$$

2.7.3 Binary Multiplication

The four basic rules for multiplying bits are as follows:

$$0 \times 0 = 0, \quad 0 \times 1 = 0, \quad 1 \times 0 = 0, \quad 1 \times 1 = 1$$

Multiplication is performed with binary numbers in the same manner as with decimal numbers. It involves forming partial products, shifting each successive partial product left one place, and then adding all the partial products.

Example: Perform the following binary multiplications:

(a) $11_2 \times 11_2$ (b) $101_2 \times 111_2$

Sol.

(a)	$\begin{array}{r} 11 \\ \times 11 \\ \hline 11 \\ +11 \\ \hline 1001 \end{array}$	(b)	$\begin{array}{r} 111 \\ \times 101 \\ \hline 111 \\ 000 \\ +111 \\ \hline 100011 \end{array}$
	{ Partial products		{ Partial products

2.7.4 Binary Division

Division in binary follows the same procedure as division in decimal

Example: Perform the following binary divisions: (a) $110_2 \div 11_2$ (b) $110_2 \div 10_2$

Sol.

(a)	$\begin{array}{r} 10 \\ 11 \overline{)110} \\ \underline{11} \\ 000 \end{array}$	(b)	$\begin{array}{r} 11 \\ 10 \overline{)110} \\ \underline{10} \\ 10 \\ \underline{10} \\ 00 \end{array}$
	$\begin{array}{r} 2 \\ 3 \overline{)6} \\ \underline{6} \\ 0 \end{array}$		$\begin{array}{r} 3 \\ 2 \overline{)6} \\ \underline{6} \\ 0 \end{array}$



First Semester Exam for 2022-2023

Answer all the Following Questions

**Q1\ define number system and explain each type of it with their range numbers.
(20 marks)**

Q2\ convert the following numbers to base 10 for five only. (25 marks)

Type of systems	numbers
Binary	010101010
	000111000
octal	67
	21
hexadecimal	7E3D
	8210

Q3\ solve all operations below (30 marks)

A\ addition the following binary number.

$$\begin{array}{r} 1) \quad 111 \quad 2) \quad 1001 \\ + 011 \quad + 1100 \end{array}$$

B\ subtraction the following binary number.

$$\begin{array}{r} 1) \quad 110 \quad 2) \quad 110 \\ - 100 \quad - 011 \end{array}$$

C\ multiplication the following binary number.

$$\begin{array}{r} 1) \quad 11 \quad 2) \quad 11 \quad 3) \quad 1010 \\ \times 11 \quad \times 0111 \quad \times 0101111 \end{array}$$

D\ division the following binary number.

$$\begin{array}{r} 1) \quad 10 \quad 2) \quad 11 \quad 3) \quad 101 \\ \div 110 \quad \div 0111 \quad \div 1111 \end{array}$$

Q4\ Simplify the following expression as below (25 marks)

$$(A+B).(A+C) = A + BC$$

Good luck