In mathematics, Fourier Series is a method that allows any Periodic mathematical function to be written as a Series or a Combination of Sin and Cosine functions multiplied by a certain Parameter.

Basic of Integ

① $\int a\,dx \Rightarrow ax + c$ ex/ $\int 3\,dx = 3x + c$

② $\int x^n dx \Rightarrow \frac{x^{n+1}}{n+1} + c$ ex/ $\int x^2 dx = \frac{x^3}{3} + c$

③ $\int \sin x\,dx = -\cos x + c$ ex/ $\int \sin 3x\,dx = \frac{1}{3}\int \sin 3x \cdot 3\,dx = \frac{1}{3}[-\cos 3x] = -\frac{1}{3}\cos 3x + c$

④ $\int \cos x\,dx = \sin x + c$ ex/ $\int \cos 2x \cdot 2\,dx = \sin 2x + c$

⑤ $\int x^2 \sin x\,dx$ تكامل بالتجزئة او الاختزال

$x^2 \;\oplus\; \sin x$
$2x \;\ominus\; -\cos x$
$2 \;\oplus\; -\sin x$
$0 \qquad +\cos x$

$x^2\cos x + 2x\sin x + 2\cos x + c$

$\int (\pi - x)\cos x\,dx$

$\pi - x \;\oplus\; \cos x$
$-1 \;\ominus\; \sin x$
$0 \qquad -\cos x$

$(\pi - x)\sin x - \cos x + c$

كيف نجد Period

Find the period ?

Period $P_{big} - P_{small}$

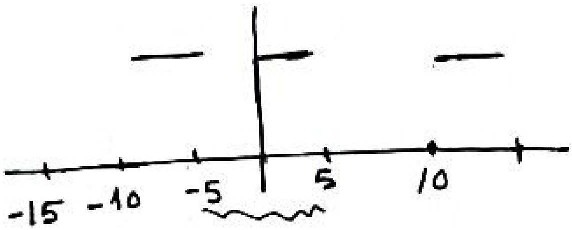
Ex

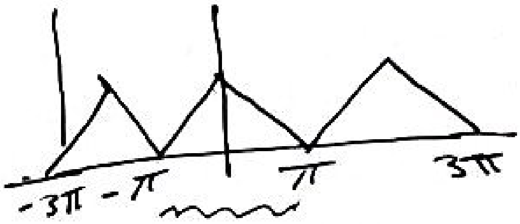
① $f(x) = x^2 \quad 0 < x < \pi$, $P = \pi - 0 = \pi$

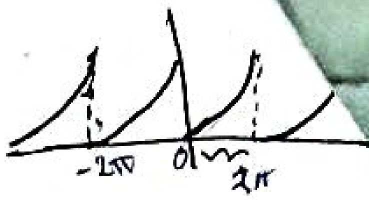
② $f(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases}$ $P = 5 - (-5) = 10$

③ $f(x) = \begin{cases} -x & -1 < x < 0 \\ x & 0 < x < 1 \\ 1 & 1 < x < 3 \end{cases}$ $P = 3 - (-1) = 3 + 1 = 4$

② Find the priode ?







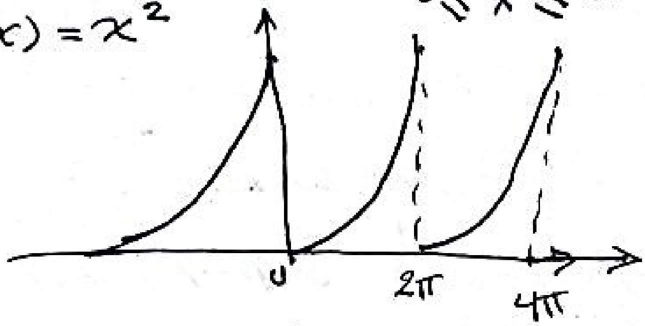
$P = 2\pi - 0$
$= 2\pi$

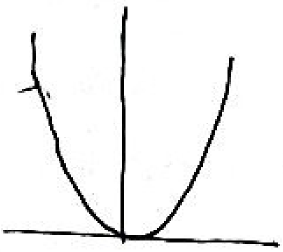
$P = \pi - (-\pi)$
$= 2\pi$

~~[illegible]~~
$P = 5 - (-5) = 10$

Draw the function ?

① $f(x) = x^2$ $0 \leq x \leq 2\pi$



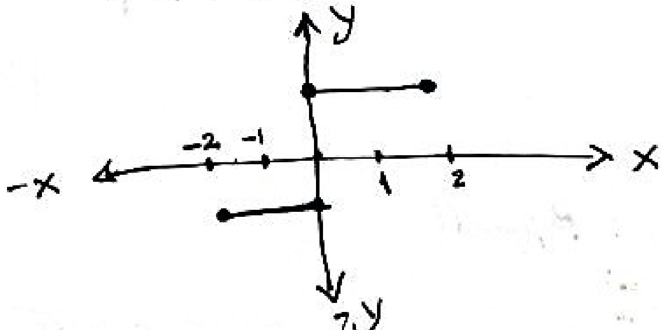


orginal

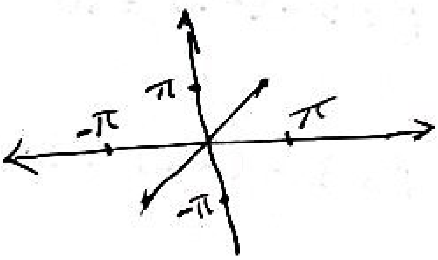
② $f(x) = \begin{cases} -1 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$

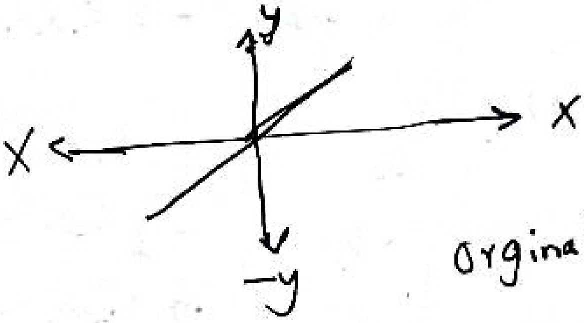
Sol:-

$f(x) = \begin{cases} -1 & -2 < x < 0 \\ 1 & 0 < x < 2 \end{cases}$ $(0,-1)\ (-2,-1)$
$(0,1)\ (2,1)$



③ $f(x) = x$ $-\pi < x < \pi$ $(-\pi, -\pi)$, (π, π)





Orginal

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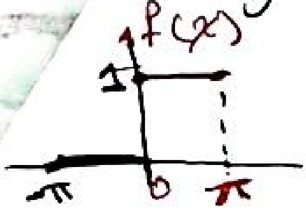
during drawing Find the function?



Sol:-

$$f(x) = \begin{cases} 1 & 0 < x < \pi \\ 0 & -\pi < x < 0 \end{cases}$$

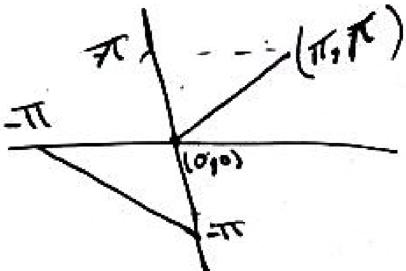
$(0,1)$, $(\pi,1)$

$(-\pi,0)$, $(0,0)$

(2)

$f(x) = x^2 \qquad 0 < x < 2\pi$

(3)



$$f(x) = \begin{cases} x & (0,0),\ (\pi,\pi) \\ -\pi - x & (-\pi,0)\ (0,-\pi) \end{cases}$$

Notes:-

(1) $\sin n\pi = 0$

(2) $\cos n\pi = (-1)^n$

(3) $\sin 2n\pi = 0$

(4) $\cos 2n\pi = 1$

(5) $$\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & n = \text{even} \\ 1 & n = 1, 5, 9 \\ -1 & n = 3, 7, 11 \end{cases} \qquad \begin{cases} (-1)^{(n-1)/2} & \text{odd} \\ 0 & n = \text{even} \end{cases}$$

(6) $$\cos\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & n = \text{odd} \\ 1 & n = 4, 8, 12 \\ -1 & 2, 6, 10 \end{cases} \qquad \begin{cases} (-1)^{n/2} & n = \text{even} \\ 0 & \text{odd} \end{cases}$$

(7) $\cos(-x) = \cos(x)$

(8) $\sin(-x) = -\sin(x)$

(9) $\sin nx \sin mx \qquad n, m = 1, 2, 3 \ldots$

$$= \frac{1}{2}\left[\cos(n-m)x - \cos(n+m)x\right]$$

(10) $\cos nx \cos mx \qquad n, m = 1, 2, 3 \ldots$

$$= \frac{1}{2}\left[\cos(n-m)x + \cos(n+m)x\right]$$

(11) $\sin nx \cos mx$, $n, m = 1, 2 \dots$

$= \frac{1}{2}\left[\sin(n-m) + \sin(n+m)\right]$

Basic equations in Fourier series.

$$\boxed{f(x) = A + \sum_{n=1}^{\infty} a_n \cos n\omega_0 x + b_n \sin n\omega_0 x}$$

$\omega_0 = 2\pi f$, $f = \frac{1}{T} = \frac{1}{\text{Period}} = \frac{1}{P}$

$\omega_0 = 2\pi \times \frac{1}{P} = \frac{2\pi}{P}$ (P = Periode)

• $A = \frac{1}{2} a_0 = \frac{1}{2} \times \frac{1}{\text{half period}} \int f(x)\, dx$ —— (1)

(big)

• $a_n = \frac{1}{\text{Half Period}} \int f(x) \cos n\omega_0 x \, dx$ —— (2)

(Small)

(• $b_n = \frac{1}{\text{Half period}} \int f(x) \sin n\omega_0 x \, dx$ —— (3)

Half period

$P = 4$, half period $= 2$

$P = 2\pi$, " " " $= \pi$

Ex1 Find Fourier Series

$$f(x) = \begin{cases} 1 & 0 \le x \le \pi \\ 0 & -\pi \le x \le 0 \end{cases}$$

Sol:-

$P = \pi - (-\pi) = 2\pi$, half priode $= \frac{2\pi}{2} = \pi$

$\omega_0 = \frac{2\pi}{P} = \frac{2\pi}{2\pi} = 1$

$f = A + \sum_{n=1}^{\infty} a_n \cos n\omega_0 x + b_n \sin n\omega_0 x$

• $A = \frac{1}{2} a_0 = \frac{1}{2} \times \frac{1}{\text{half } P} \int_{-\pi}^{\pi} f(x)\, dx$

$= \frac{1}{2} \times \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)$

$\frac{1}{2\pi}\left[\int_0^{\pi} 1\,dx + \cancel{\int_{-\pi}^{0} 0\,dx}\right]$ المدرس المادة / اوس الشاهر

$= \frac{1}{2\pi}[x]_0^{\pi} = \frac{1}{2\pi}[\pi - 0] = \frac{\pi}{2\pi} = \frac{1}{2}$

Zero

- $a_n = \frac{1}{h.p}\int f(x)\cos n w_o x\,dx = \frac{1}{\pi}\left[\int_0^{\pi} 1 \cos nx\,dx + \cancel{\int_{-\pi}^{0}(0)\cos nx\,dx}\right]$

$a_n = \frac{1}{\pi}\int_0^{\pi}\cos nx\,dx = \frac{1}{n\pi}\int_0^{\pi}\cos nx * n\,dx$

$= \frac{1}{n\pi}[\sin nx]_0^{\pi} = \frac{1}{n\pi}[\sin n\pi - \sin n*0]$

$= \frac{1}{n\pi}[\cancel{\sin n\pi}] = 0$ (Zero)

$b_n = \frac{1}{h.p}\int f(x)\sin n w_o x\,dx = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$= \frac{1}{\pi}\left[\int_0^{\pi} 1 \sin nx\,dx + \cancel{\int_{-\pi}^{0} 0 \sin nx\,dx}\right]$

$= \frac{1}{\pi}\int_0^{\pi}\sin nx\,dx = \frac{1}{n\pi}\int_0^{\pi}\sin nx * n\,dx$

$= \frac{1}{n\pi}[-\cos nx]_0^{\pi} = \frac{1}{n\pi}[-\cos\pi n - (-\cos 0)]$

$= \frac{1}{n\pi}[-\cos\pi n + 1] = \frac{1}{n\pi}[-(-1)^n + 1]$

Sub. $n = 1, 2$

$n = 1$, $b_1 = \frac{1}{1*\pi}[-(-1)^1 + 1] = \frac{1}{\pi}[1 + 1] = \frac{2}{\pi}$

$= \frac{2}{n\pi} = b_n$

$n = 2$, $b_2 = \frac{1}{2\pi}[-(-1)^2 + 1] = \frac{1}{2\pi}[\cancel{-1+1}] = 0$ (Zero)

Zero

$f(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left[\cancel{(0)\cos n w_o x} + \frac{2}{n\pi}\sin n w_o x\right]$

$f(x) = \frac{1}{2} + \sum_{n=odd}^{\infty}\frac{2}{n\pi}\sin nx$ Sub in odd $(1, 3, 5, \dots)$

$f(x) = \frac{1}{2} + \left(\frac{2}{\pi}\sin x + \frac{2}{3\pi}\sin 3x + \dots\right)$

(odd , even)

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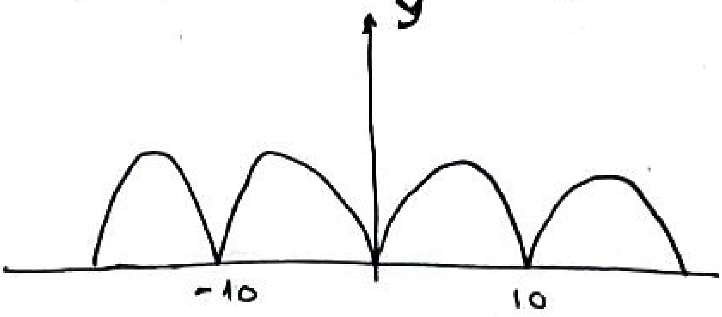
eve

even

$b_n = 0$

تحسب

A, a_n



odd

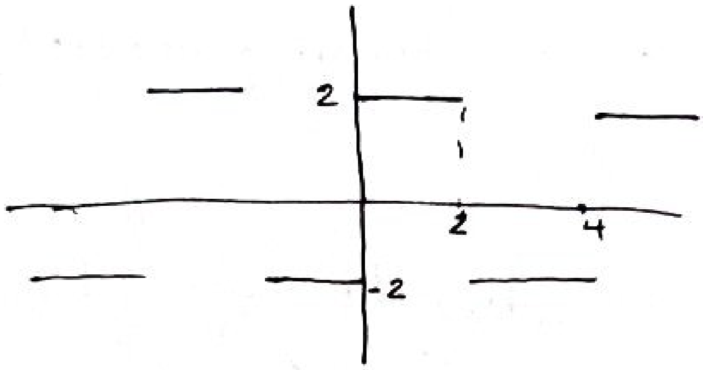
odd

$A = 0$

$a_n = 0$

فقط تحسب

b_n



even , A

$$a_n = \frac{1}{H.P} * 2 \int f(x) \cos n\omega_0 x \, dx$$

odd

$$b_n = \frac{1}{H.P} * 2 \int f(x) \sin n\omega_0 x \, dx$$

Ex

$$f(x) \begin{cases} -1 & -1 < x < 0 \\ 1 & 0 < x < 1 \end{cases}$$

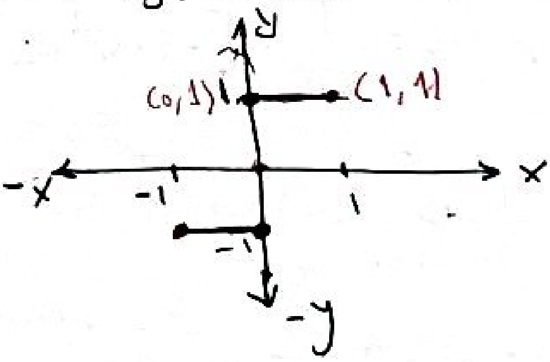
Find Fourier Series and what type this function?

x, y　　x, y

$(-1, -1)$, $(0, -1)$

$(0, 1)$, $(1, 1)$

odd function



$A = 0$

$a_n = 0$

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$$= \frac{1}{h.P} * 2 \int f(x) \sin n\omega x \, dx$$

half priod

$$P = 1-(-1) = 2 \;,\; h.P = \frac{2}{2} = 1$$

$$\omega = \frac{2\pi}{P} = \frac{\cancel{2}\pi}{\cancel{2}} = \pi$$

$$\therefore b_n = \frac{1}{1} * 2 \int 1 * \sin n\pi x \, dx = 2 * \frac{1}{\pi n} \int_0^1 \sin \pi n x \, dx \; *\pi n$$

$$b_n = \frac{2}{\pi n} \int_0^1 \sin \pi n x * \pi n \, dx$$

$$= \frac{2}{\pi n} \left[-\cos \pi n x \right]_0^1$$

$$= \frac{2}{\pi n} \left[-\cos \pi n - (-\cos \pi n * 0) \right] = \frac{2}{\pi n} \left[-\cos \pi n + \cos 0 \right)$$

$$= \frac{2}{\pi n} \left[-\cos \pi n + 1 \right] = \frac{2}{\pi n} \left[-(-1)^n + 1 \right]$$

n = 1, n = 2

$$= \frac{2}{\pi n} \left[-(-1)^n + 1 \right] \begin{cases} 0 & n = \text{even} \\ \frac{4}{\pi n} & n = \text{odd} \end{cases}$$

$$f(x) = \overset{\text{zero}}{\cancel{A}} + \sum_{n=1}^{\infty} \overset{\text{zero}}{\cancel{a_n \cos n\omega_0 x}} + b_n \sin n\omega_0 x$$

$$f(x) = \sum_{n=odd}^{\infty} b_n \sin n\pi x \qquad , n = 1, 3, 5$$

$$= \sum_{n=odd}^{\infty} \frac{4}{\pi n} \sin n\pi x$$

$$\therefore f(x) = \underbrace{\frac{4}{\pi} \sin \pi x}_{n=1} + \underbrace{\frac{4}{3\pi} \sin 3\pi x}_{n=3} + \underbrace{\frac{4}{5\pi} \sin 5\pi x}_{n=5}$$