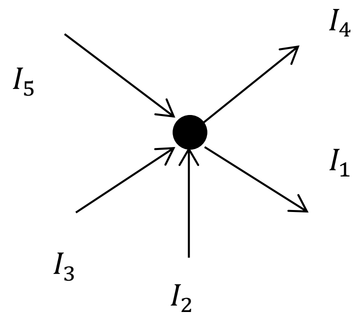
16 : Kirchhoff's laws

First law: The algebraic sum of the currents flowing toward a node is zero.

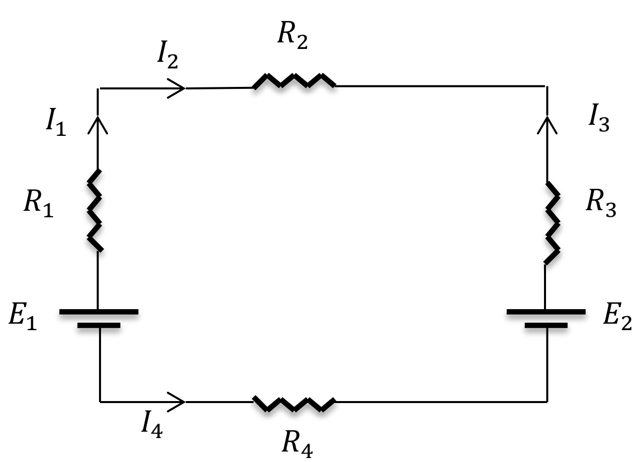
$$I_2 + I_3 + I_5 - I_1 - I_4 = 0$$



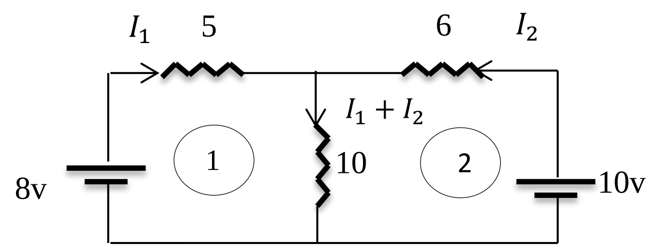
Second law: In a closet circuit the algebraic sum of the products of the current and the resistance of each part of the circuit is equal to the resultant e.m.f in the circuit.

$$\boldsymbol{I_1R_1 + I_2R_2 - I_3R_3 - I_4R_4 = E_1 - E_2}$$

$$\boldsymbol{I_1R_1 + I_2R_2 - I_3R_3 + E_2 - I_4R_4 - E_1 = 0}$$



EX.1: For the circuit shown below calculate currents I_1, I_2, I_3 using Kirchhoff's laws.



Solution:-

loop 1

$$5I_1 + 10(I_1 + I_2) = 8$$

$$5I_1 + 10I_1 + 10I_2 = 8$$

$$15I_1 + 10I_2 = 8 \quad \dots \dots \dots (1)$$

loop 2

$$6I_2 + 10(I_1 + I_2) = 10$$

$$6I_2 + 10I_1 + 10I_2 = 10$$

$$10I_1 + 16I_2 = 10 \dots \dots \dots (2)$$

Using matrix form

$$\begin{bmatrix} 15 & 10 \\ 10 & 16 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \end{bmatrix} \rightarrow \Delta R = \begin{vmatrix} 15 & 10 \\ 10 & 16 \end{vmatrix} = 140$$

$$\Delta I_1 = \begin{vmatrix} 8 & 10 \\ 10 & 16 \end{vmatrix} = 28 \rightarrow \therefore I_1 = \frac{\Delta I_1}{\Delta R} = \frac{28}{140} = 0.2A$$

$$\Delta I_2 = \begin{vmatrix} 15 & 8 \\ 10 & 10 \end{vmatrix} = 70 \rightarrow \therefore I_2 = \frac{\Delta I_2}{\Delta R} = \frac{70}{140} = 0.5A$$

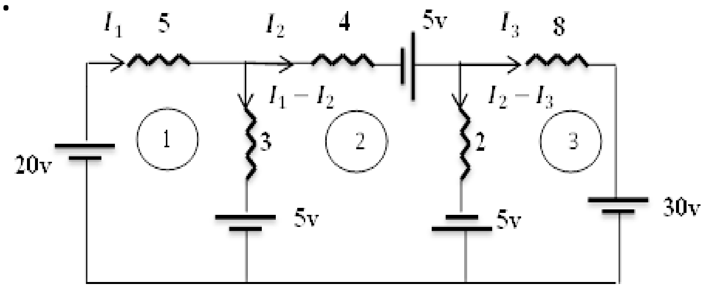
$$I_3 = I_1 + I_2 = 0.2 + 0.5 = 0.7A$$

Verify:

$$15I_1 + 10I_2 = 8$$

$$(15 * 0.2) + (10 * 0.5) = 8$$

<u>EX.2:</u> By applying Kirchhoff's laws, calculate branch currents in the network below.



<u>Solution:-</u>

<u>loop 1</u>

$$5I_1 + 3(I_1 - I_2) = 20 - 5$$

$$5I_1 + 3I_1 - 3I_2 = 15$$

$$8I_1 - 3I_2 = 15 \dots \dots \dots (1)$$

<u>loop 2</u>

$$4I_2 + 2(I_2 - I_3) - 3(I_1 - I_2) = 5 + 5 + 5$$

$$4I_2 + 2I_2 - 2I_3 - 3I_1 + 3I_2 = 15$$

$$-3I_1 + 9I_2 - 2I_3 = 15 \dots \dots \dots (2)$$

<u>loop 3</u>

$$8I_3 - 2(I_2 - I_3) = -30 - 5$$

$$8I_3 - 2I_2 + 2I_3 = -35$$

$$-2I_2 + 10I_3 = -35 \dots \dots \dots (3)$$

Put in matrix form:

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

$$\Delta R = 598$$

$$\Delta I_1 = \begin{bmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{bmatrix} = 1530$$

$$\therefore I_1 = \frac{\Delta I_1}{\Delta R} = \frac{1530}{598} = 2.558[A]$$

$$\Delta I_2 = \begin{bmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{bmatrix} = 1090$$

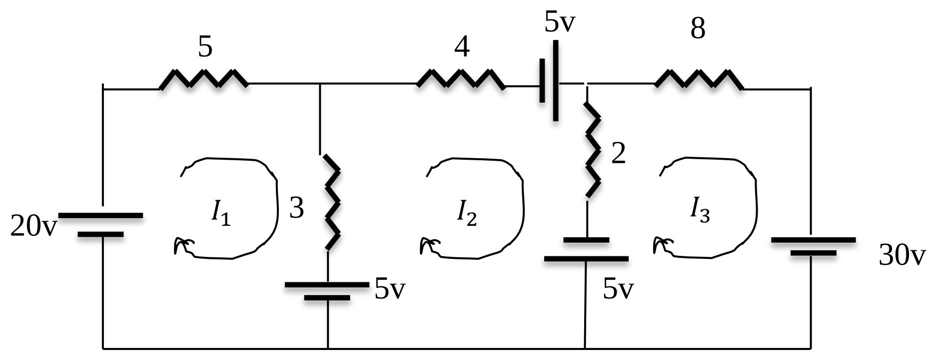
$$\therefore I_2 = \frac{\Delta I_2}{\Delta R} = \frac{1090}{598} = 1.822[A]$$

$$\Delta I_3 = \begin{bmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{bmatrix} = -1875$$

$$\therefore I_3 = \frac{\Delta I_3}{\Delta R} = \frac{-1875}{598} = -3.135[A]$$

\\Since I_3 has a negative sign, hence, its direction is opposite to the assumed direction.

17: Maxwell's current loops



Loop (1)

$$8I_1 - 3I_2 = 15 \quad \dots \dots \dots \dots (1)$$

Loop (2)

$$9I_2 - 3I_1 - 2I_3 = 15$$

$$-3I_1 + 9I_2 - 2I_3 = 15 \dots \dots \dots \dots (2)$$

Loop (3)

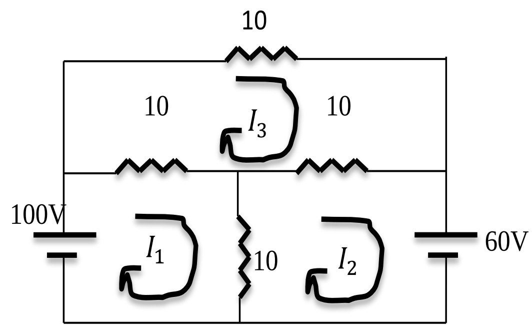
$$10I_3 - 2I_2 = -35$$

$$-2I_2 + 10I_3 = -35 \dots \dots \dots \dots (3)$$

$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

Then complete the solution as in last example.

EX.1: for the circuit shown below, write down a set of simultaneous equation to find $I_1, I_2\ and\ I_3$ by using Maxwell's current loops.



Solution

Loop (1) $20I_1 - 10I_2 - 10I_3 = 100\ \dots\dots\dots\dots (1)$

Loop (2) $20I_2 - 10I_1 - 10I_3 = -60$

$$-10I_1 + 20I_2 - 10I_3 = -60\ \dots\dots\dots\dots (2)$$

Loop (3) $30I_3 - 10I_2 - 10I_1 = 0$

$$-10I_1 - 10I_2 + 30I_3 = 0\ \dots\dots\dots\dots (3)$$

$$\begin{bmatrix} 20 & -10 & -10 \\ -10 & 20 & -10 \\ -10 & -10 & 30 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ -60 \\ 0 \end{bmatrix}$$

Homework: using Maxwell's current loops, calculate circuit currents I_1, I_2, I_3.