22: superposition Theorem

In a linear network containing more than one source, the resultant current in any branch is the algebraic sum of the currents that would be produced by each source acting alone, all other sources being replaced by their internal resistances.

في الشبكة الخطية التي تحتوي على اكثر من مصدر للطاقة، تكون محصلة التيار في أي فرع تساوي المجموع الجبري للتيارات الناتجة عن كل مصدر عندما يؤثر لوحده في الدائرة بعد استبدال المصادر الاخرى بمقاوماتها الداخلية.

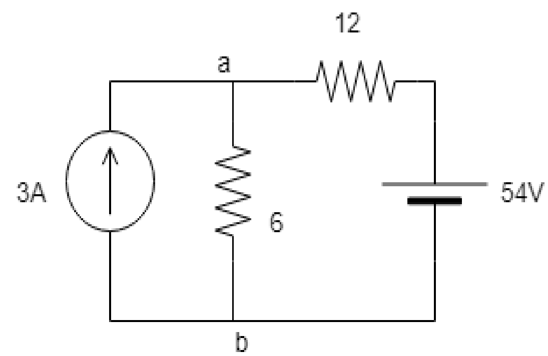
خطوات تطبيق النظرية:

١- نرفع جميع المصادر ونترك مصدر واحد في الدائرة ثم نحسب التيار المار في الفرع المطلوب بتأثير ذلك المصدر فقط مع استبدال المصادر الاخرى بمقاوماتها الداخلية، ويجب التأكد من اتجاه التيار في الفرع المطلوب.

٢- نعيد المصدر المرفوع في الخطوة(١) اعلاه ثم نرفع المصدر الثاني ونحسب مقدار التيار ونحدد اتجاهه، وهكذا نكرر العملية لجميع المصادر.

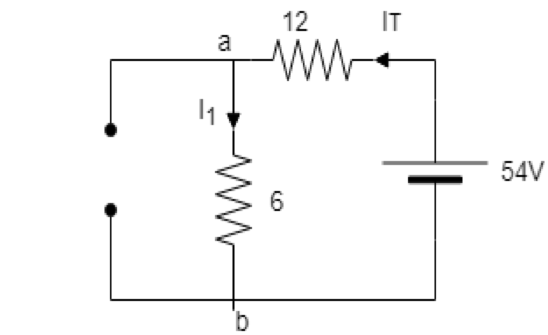
٣- نجمع التيارات الناتجة جمعاً جبرياً فيكون الناتج يمثل التيار النهائي المار في ذلك الفرع.

EX: for the circuit shown below, find the current passing in the (6Ω) resistor using superposition theorem.



Solution:

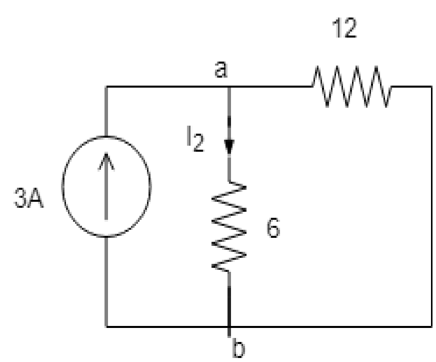
1- Consider voltage source acting alone



$$I_T = \frac{54V}{12+6} = 3A$$

$$I_1 = I_T = 3A$$

2- Consider current source acting alone.



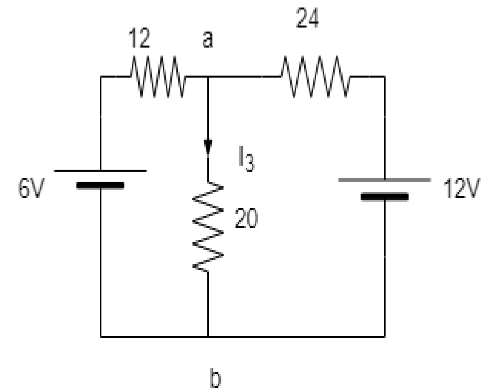
$$I_2 = 3 \times \frac{12}{12+6} = 2A$$

$\therefore$ *total current in the* 6Ω *resistor*

$$I_1 + I_2 = 3 + 2 = 5A$$

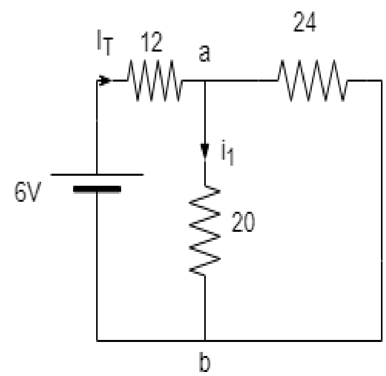
ملاحظة: عندما تكون اتجاهات التيارات متعاكسة في المقاومة المطلوب حساب التيار فيها فيجب طرح التيارات من بعضها وليس جمعها.

EX: using superposition theorem calculate the current (I3) in the circuit below.



Solution:

1-consider the (6V) source acting alone:



$$R_{T_1} = \frac{24 \times 20}{24 \times 20} + 12$$

$$= 10.9 + 12 = 22.9\ \Omega$$

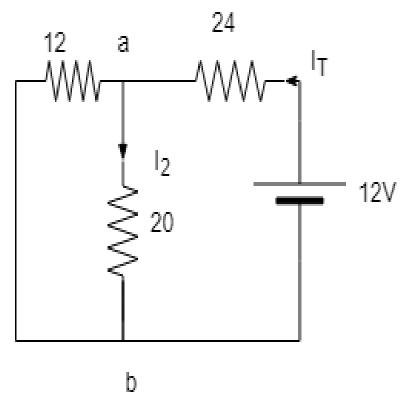
$$I_T = \frac{6}{22.9} = 0.262\ A$$

$$I_1 = I_T \times \frac{24}{24 + 20}$$

$$= 0.262\ \frac{24}{24 + 20}$$

$$= 0.1429\ A,\ \ from\ a\ to\ b$$

2- Consider the 12V supply acting alone:-



$$R_{T_2} = \frac{12 \times 20}{12 \times 20} + 24 = 7.5 + 24 = 31.5\Omega$$

$$I_T = \frac{12}{31.5} = 0.38\ A$$

$$I_2 = I_T \frac{12}{12 + 20} = 0.38 \frac{12}{32} = 0.142\ A$$

$$I_3 = I_1 + I_2 = 0.1429 + 0.142 = 0.2849\ A$$

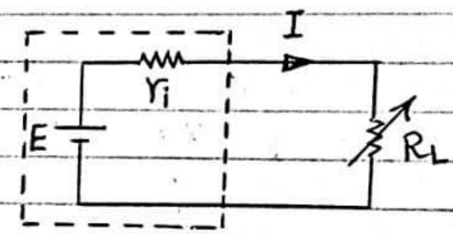
23: Maximum power Transfer

Maximum output power is obtained from a network or source when the load Resistance (R_L) is equal to the internal resistance (r_i) of the source.

$$\boxed{R_L = r_i}$$

- Derivation of formula

Consider a source having an e.m.f (E), and internal resistance (r_i). We will show that to obtain maximum power from the source, the Load resistance R_L must be equal to the source resistance r_i



$$I = \frac{E}{R_L + r_i} \quad ---- \ (1)$$

$$V_{R_L} = I \cdot R_L = \frac{E}{R_L + r_i} \cdot R_L$$

$$P_L = V_{R_L} \cdot I \quad ---- \ (2)$$

$$= \frac{E \cdot R_L}{R_L + r_i} \cdot \frac{E}{R_L + r_i}$$

$$= \frac{E^2 \cdot R_L}{(R_L + r_i)^2} \quad ---- \ (3)$$

To find the value of R_L at which maximum power is obtained we differentiate equation (3) with respect to R_L then equating the derivative to zero.

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L}\left[\frac{E^2 \cdot R_L}{(R_L + r_i)^2}\right]$$

$$= \frac{(R_L+r_i)^2 \cdot E^2 - E^2 R_L * 2(R_L+r_i)}{(R_L+r_i)^4}$$

$$= E^2 \cdot \frac{(R_L+r_i)^2 - 2R_L(R_L+r_i)}{(R_L+r_i)^4}$$

$$0 = E^2 \cdot \frac{(R_L+r_i)^2 - 2R_L(R_L+r_i)}{(R_L+r_i)^4}$$

$$0 = E^2 \left[(R_L+r_i)^2 - 2R_L(R_L+r_i)\right]$$

$$(R_L+r_i)^{\cancel{2}} = 2R_L\cancel{(R_L+r_i)}$$

$$R_L + r_i = 2R_L$$

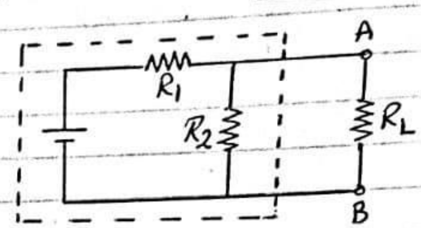
$$r_i = 2R_L - R_L$$

$$\therefore \boxed{R_L = r_i}$$

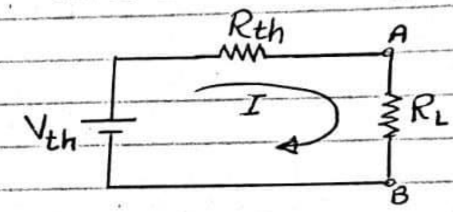
$\therefore$ For maximum power transfer load resistance (R_L) equals internal source resistance (r_i)

For a linear d.c. network, the load will receive maximum power from the network when its total resistive value is equal to the Thevenin resistance of the network as "seen" by the load, $\boxed{R_L = R_{th}}$

To derive an expression for maximum power $P_{L(max)}$ we proceed as follows:–



$$I = \frac{V_{th}}{R_{th} + R_L}$$



$$P_L = I^2 \cdot R_L$$

$$= \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 \cdot R_L$$

$$\therefore P_L = \frac{V_{th}^2 \cdot R_L}{(R_{th} + R_L)^2}$$

for maximum power $\boxed{R_L = R_{th}}$

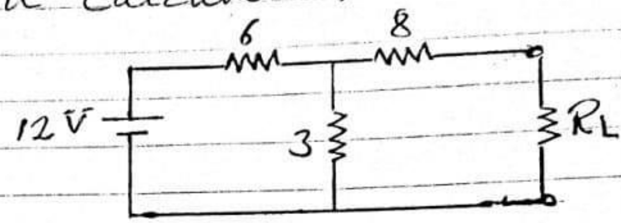
$$\therefore P_{L(max)} = \frac{V_{th}^2}{(R_{th} + R_{th})^2} \cdot R_{th}$$

$$= \frac{V_{th}^2}{(2R_{th})^2} \cdot R_{th}$$

$$\therefore P_{L(max)} = \frac{V_{th}^2}{4R^2} \cdot \cancel{R_{th}} \Rightarrow \boxed{P_{L(max)} = \frac{V_{th}^2}{4R}}$$

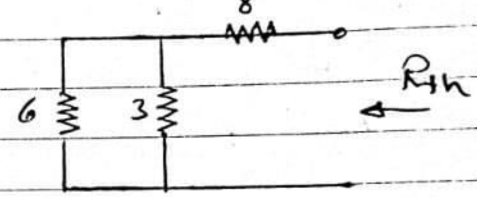
$$P_{L(\max)} = \frac{V_{th}^2}{4R_{th}^2} \cdot R_{th} \rightarrow P_{L(max)} = \frac{V_{th}^2}{4R_{th}}$$

Ex for the circuit shown below, find the value of R_L for maximum power transfer, then calculate the maximum power $P_{L(max)}$.



Solution:-

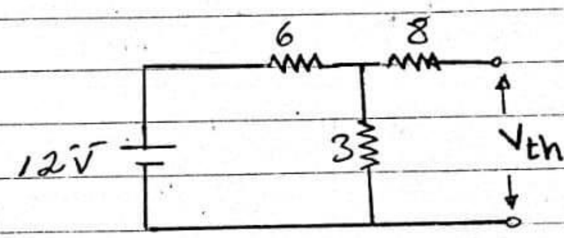
1- find R_{th}



$$R_{th} = \frac{6 \times 3}{6+3} + 8 = 10\,\Omega$$

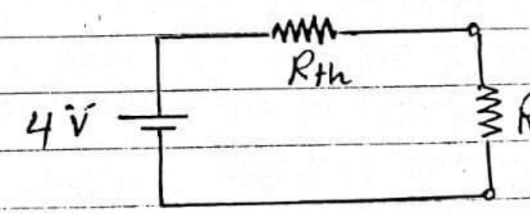
∴ for maximum power transfer

$$\boxed{R_L = R_{th}} = 10\,\Omega$$



② find V_{th}

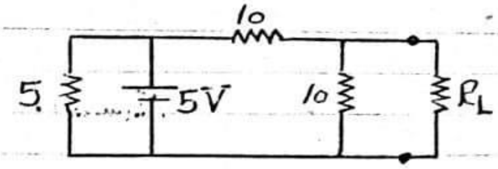
$$V_{th} = 12 \times \frac{3}{3+6} = 4\,V$$



$$\boxed{P_{L(max)} = \frac{V_{th}^2}{4 R_{th}}}$$

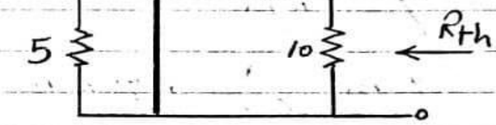
$$= \frac{4^2}{4 \times 10} = \underline{0.4\ W}$$

EX: Using maximum power transfer theorem, find the value of load resistance (R_L) required to obtain maximum power at load, then calculate the maximum power $P_{L(max)}$.



Solution:-

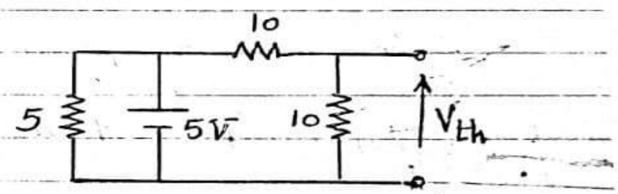
1- find R_{th}



$$R_{th} = \frac{10 \times 10}{10 + 10} = 5\,\Omega$$

$$\therefore R_L = R_{th} = 5\,\Omega$$

2- find V_{th}



$$V_{th} = 5V \times \frac{10}{10+10} = 2.5V$$

3- Calculate $P_{L(max)}$

$$P_{L(max)} = \frac{V_{th}^2}{4R_{th}} = \frac{(2.5)^2}{4 \times 5} = 0.3125\,[W]$$

H.W: for the circuit shown below, calculate the load resistance R_L for maximum power transfer, then calculate the maximum power $P_{L(max)}$.