دوائر كهربائية\ المحاضرة الاولى

Systems of UNITS

Quantity	Basic Unit	Symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic Temperature	kelvin	K
Luminous intensity	candela	cd

The SI Prefixes

Multiple	Prefix	Symbol
10^9	giga	G
10^6	mega	M
10^3	kilo	k
10^{-3}	milli	m
10^{-6}	micro	µ
10^{-9}	nano	n
10^{-12}	pico	p

The derived unite commonly used in electric circuit theory.

Quantity	Unit	Symbol
Electric charge	Coulomb	C
Electric potential	Volt	V
Resistance	Ohm	Ω
Conductance	Simens	S
Inductance	Henry	H
Capacitance	Farad	F
Frequency	Hertz	Hz
Power	Watt	W
Magnetic flux	Weber	Wb
Magnetic flux density	Tesla	T
Force	Newton	N
Energy, work	Joule	J

Greek letters used as symbols

α	Alpha	λ	Lambda
β	Beta	μ	MU
Δ(δ)	Delta	π	Pi
ε	Epsilon	ρ	Rho
η	Eta	σ (Σ)	Sigma
Θ	theta		

Definition

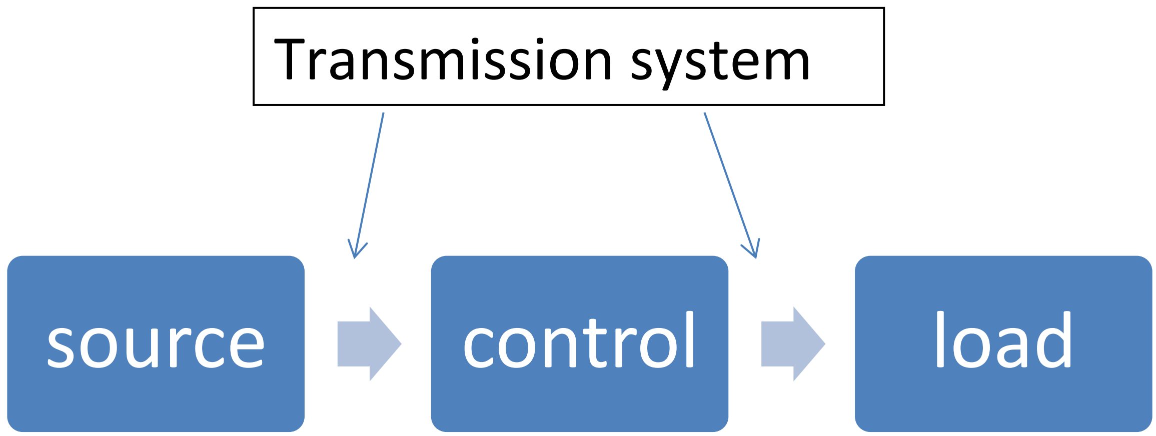
Ohm: It is the unit of resistance, defined as:
The resistance which permits a current of (1A) to flow when a potential difference of (1V) is applied to the resistance.

Volt: It is the unit of e.m.f and potential difference, defined as: The potential difference between two points on a conductor carrying a constant current of (1A) when the power dissipated between these points is (1W).

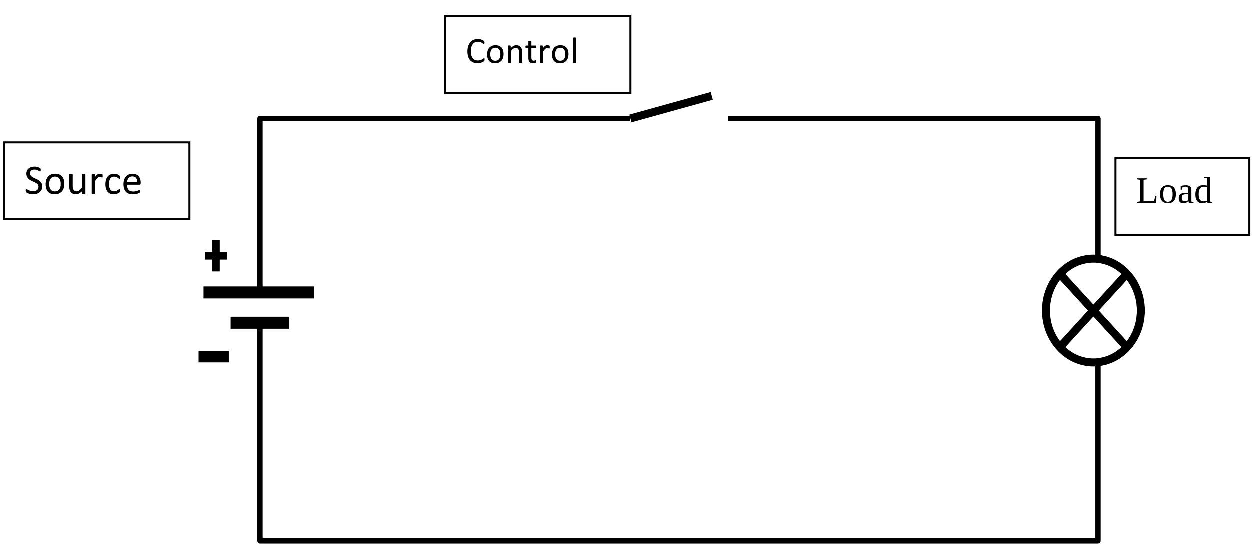
Ampere: It is the unit of electric current, defined as:
That current when flowing in two long parallel conductors 1 meter apart exerts a force of 2×10^{-7} newton per meter on each conductor.

Electric circuit components

Electric circuit: - Any closed path maintains the flow of electric current.



simple electric circuit



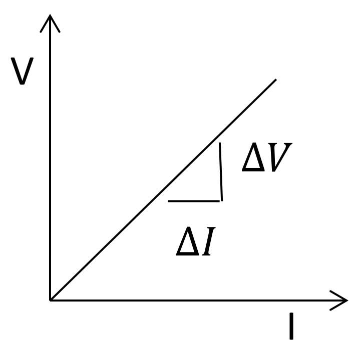
Ohm's Law:-

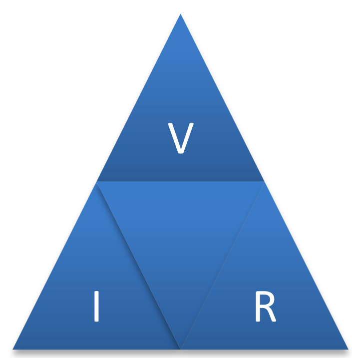
The ratio of potential difference (V) between any two points on a conductor to the current (I) flowing between them is a constant, provided temperature does not change.

$$\frac{V}{I} = constant \rightarrow \frac{V}{I} = R$$

$$\therefore I \propto V \rightarrow V = IR$$

This implies a liner relationship.





$$V = I.R\ ,\ I = \frac{V}{R}\ ,R = \frac{V}{I}$$

* Power (P) is the time rate of doing work.

Power (P) =Voltage × current [Watt]

$$P = V.I$$

$$P = I^2.R$$

$$P = \frac{V^2}{R}$$

EX1: In the circuit shown below, find the value of resistance required to maintain a current of (2A) in the circuit.

Solution:

Using ohm's law;

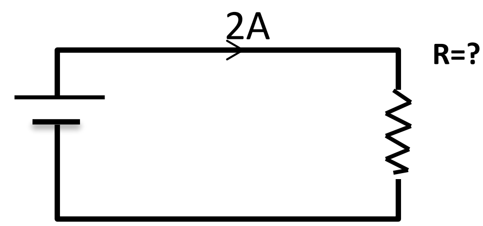
$V = I.R \rightarrow R = \frac{V}{I}$

$$\therefore R = \frac{20V}{2A} = 10\Omega$$

EX2: In the circuit shown below if the power dissipated in the resistor was (10W), find the value of resistance and the circuit e.m.f (volts).

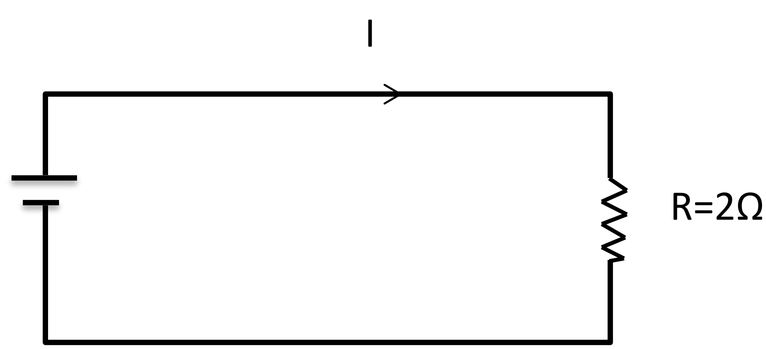
Solution: $\mathrm{P = E.I}$

$= V.I \rightarrow V = \frac{P}{I} = \frac{10W}{2A} = 5\Omega$



$$using\ ohm'slaw; R = \frac{V}{I} = \frac{5}{2} = 2.5\Omega$$

H.W: If the voltage across the resistance is (20mV), find the value of current and the power delivered from the battery.



Resistance of conductors مقاومة الموصلات

It is a measure of the opposition offered by a conductor to the flow of electric current.

The resistance of conductor depends on the following factors:-

- It varies directly as length 'L' ….. $R \propto L$
- It varies in inversely as cross sectional area …. $R \propto \frac{1}{A}$
- It depends on nature of material denoted by(ρ) specific resistance or resistivity.
- It also depends on temperature of conductors

  ∴ we can relate the resistance to the first three factors by the following relationship:-

$$R = \rho . \frac{L}{A}$$

Where R: resistance in Ω

ρ : specific resistance in [Ω.m]

L: length of conductors in [m]

A: cross – sectional area in [m^2]

Specific resistance (resistivity) : It is the resistance between the opposite faces of meter cube of material.

$$since\ R = \rho . \frac{L}{A} [\Omega]$$

$$\therefore \rho = \frac{R.A}{L}\ [\Omega . m]$$

Resistivity of some conductors and insulators of 20 ℃

Material	P(Ω.m) $\times 10^{-8}$	Material	P(Ω.m) $\times 10^{-8}$
Silver	1.62	**nichrom**	10^{8}
Copper	1.72	**Bakelite**	10^{10}
Gold	2.40	**Glass**	10^{12}
Aluminum	2.83	**Mica**	10^{15}
Tungsten	5.48	**Rubber**	10^{16}
Brass	8	**Shellac**	10^{14}
Platinum	10		

EX: The resistance of a conductor $1mm^2$ in cross-sectional area and 20m long is 0.346Ω .Determine the specific resistance of the conducting material.

Solution: $A = 1mm^2 = 1 \times 10^{-6} m^2$

$L = 20\ m$

R=0.346Ω

$$\therefore R = \rho.\frac{L}{A} \rightarrow \rho = \frac{R.A}{L} = \frac{0.345 \times 10^{-6}}{20}$$

$$\therefore \rho = 1.73 \times 10^{-8} [\Omega.m]$$

EX: calculate the resistance per meter length of Aluminum wire 5mm in diameter. Given that $\rho_{al} = 2.83 \times 10^{-8}$

Sol: $A = \frac{\pi D^2}{4} = \frac{\pi \times (5 \times 10^{-3})^2}{4}$

$=6.25\pi \times 10^{-6} m^2$

$$R = \rho.\frac{L}{A} = \frac{2.83 \times 10^{-8} \times 1m}{6.25\pi \times 10^{-6} m^2} = 1.44 \times 10^{-3} [\Omega]$$

Temperature coefficient of resistance

The resistance of all pure metals increases with increase of temperature, whereas the resistance of carbon, electrolytes and insulating materials decreases with increase of temperature, certain alloys, such as manganin , shown practically no change of resistance for considerable variation of temperature. The ratio of change of resistance per degree change of temperature to the resistance at some definite temperature, adopted as standard, is termed the temperature coefficient of resistance and is represented by the Greek letter (α).

*In general if a material having a resistance ($R_\circ$) at (0℃)has a resistance (R_1) at (t_1) and (R_2) at ($t_{2)}$ and if($\alpha_\circ$) is the temperature coefficient of resistance at 0℃

$$R_1 = R_\circ(1 + \alpha_\circ t_1) \quad and \quad R_2 = R_\circ(1 + \alpha_\circ t_2)$$

$$\frac{R_1}{R_2} = \frac{1 + \alpha_\circ t_1}{1 + \alpha_\circ t_2}$$

*It is often the practice to assume the standard temperature to be (20℃), this involve using a different value for the temperature coefficient.

For material having resistance (R_{20}) at (20℃) and temperature coefficient of resistance (α_{20}) at (20℃) the resistance (R_t)at temperature (t) is given by.

$$R_t = R_{20}(1 + \alpha_\circ(t - 20))$$

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Problem 3. A wire of length 8 m and cross-sectional area 3 mm^2 has a resistance of 0.16 Ω. If the wire is drawn out until its cross-sectional area is 1 mm^2, determine the resistance of the wire.

Resistance R is directly proportional to length l, and inversely proportional to the cross-sectional area, a, i.e.,

i.e., $R \propto \frac{l}{a}$ or $R = k\left(\frac{l}{a}\right)$, where k is the coefficient of proportionality.

Since $R = 0.16$, $l = 8$ and $a = 3$, then $0.16 = (k)\left(\frac{8}{3}\right)$, from which

$k = 0.16 \times \frac{3}{8} = 0.06$

If the cross-sectional area is reduced to $\frac{1}{3}$ of its original area then the length must be tripled to 3×8, i.e., 24 m

$$\text{New resistance} \quad R = k\left(\frac{l}{a}\right) = 0.06\left(\frac{24}{1}\right) = \mathbf{1.44\ \Omega}$$

Problem 4. Calculate the resistance of a 2 km length of aluminium overhead power cable if the cross-sectional area of the cable is 100 mm^2. Take the resistivity of aluminium to be 0.03×10^{-6} Ωm

Length $l = 2$ km $= 2000$ m; area, $a = 100\ mm^2 = 100 \times 10^{-6} m^2$; resistivity $\rho = 0.03 \times 10^{-6}$ Ωm

$$\text{Resistance} \quad R = \frac{\rho l}{a} = \frac{(0.03 \times 10^{-6}\ \Omega m)(2000\ m)}{(100 \times 10^{-6}\ m^2)} = \frac{0.03 \times 2000}{100}\ \Omega$$
$$= \mathbf{0.6\ \Omega}$$

Problem 5. Calculate the cross-sectional area, in mm^2, of a piece of copper wire, 40 m in length and having a resistance of 0.25 Ω. Take the resistivity of copper as 0.02×10^{-6} Ωm

$$\text{Resistance } R = \frac{\rho l}{a} \text{ hence cross-sectional area } a = \frac{\rho l}{R}$$
$$= \frac{(0.02 \times 10^{-6}\ \Omega m)(40\ m)}{0.25\ \Omega} = 3.2 \times 10^{-6}\ m^2$$
$$= (3.2 \times 10^{-6}) \times 10^{6}\ mm^2 = \mathbf{3.2\ mm^2}$$

Problem 6. The resistance of 1.5 km of wire of cross-sectional area 0.17 mm^2 is 150 Ω. Determine the resistivity of the wire.

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Resistance, $R = \frac{\rho l}{a}$

hence, resistivity $\rho = \frac{Ra}{l} = \frac{(150\ \Omega)(0.17 \times 10^{-6}\ \text{m}^2)}{(1500\ \text{m})}$

$= \mathbf{0.017 \times 10^{-6}\ \Omega m\ or\ 0.017\ \mu\Omega m}$

Problem 7. Determine the resistance of 1200 m of copper cable having a diameter of 12 mm if the resistivity of copper is $1.7 \times 10^{-8}\ \Omega$m

Cross-sectional area of cable, $a = \pi r^2 = \pi \left(\frac{12}{2}\right)^2$

$= 36\pi\ \text{mm}^2 = 36\pi \times 10^{-6}\ \text{m}^2$

Resistance $R = \frac{\rho l}{a} = \frac{(1.7 \times 10^{-8}\ \Omega\text{m})\ (1200\ \text{m})}{(36\pi \times 10^{-6}\ \text{m}^2)}$

$= \frac{1.7 \times 1200 \times 10^6}{10^8 \times 36\pi} \Omega = \frac{1.7 \times 12}{36\pi} \Omega$

$= \mathbf{0.180\ \Omega}$

Further problems on resistance and resistivity may be found in Section 3.3, problems 1 to 7, page 29.

3.2 Temperature coefficient of resistance

In general, as the temperature of a material increases, most conductors increase in resistance, insulators decrease in resistance, whilst the resistance of some special alloys remain almost constant.

The **temperature coefficient of resistance** of a material is the increase in the resistance of a 1 Ω resistor of that material when it is subjected to a rise of temperature of 1°C. The symbol used for the temperature coefficient of resistance is α (Greek alpha). Thus, if some copper wire of resistance 1 Ω is heated through 1°C and its resistance is then measured as 1.0043 Ω then $\alpha = 0.0043\ \Omega/\Omega$°C for copper. The units are usually expressed only as 'per °C', i.e., $\alpha = 0.0043$/°C for copper. If the 1 Ω resistor of copper is heated through 100°C then the resistance at 100°C would be $1 + 100 \times 0.0043 = 1.43\Omega$

Some typical values of temperature coefficient of resistance measured at 0°C are given below:

Copper	0.0043/°C	Aluminium	0.0038/°C
Nickel	0.0062/°C	Carbon	−0.000 48/°C
Constantan	0	Eureka	0.000 01/°C

(Note that the negative sign for carbon indicates that its resistance falls with increase of temperature.)

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then the resistance R_θ at temperature θ°C is given by:

$$\boldsymbol{R_\theta = R_{20}[1 + \alpha_{20}(\theta - 20)]}$$

Problem 11. A coil of copper wire has a resistance of 10 Ω at 20°C. If the temperature coefficient of resistance of copper at 20°C is 0.004/°C determine the resistance of the coil when the temperature rises to 100°C

Resistance at θ°C, $R = R_{20}[1 + \alpha_{20}(\theta - 20)]$

Hence resistance at 100°C, $R_{100} = 10[1 + (0.004)(100 - 20)]$

$$= 10[1 + (0.004)(80)]$$
$$= 10[1 + 0.32]$$
$$= 10(1.32) = \mathbf{13.2\ \Omega}$$

Problem 12. The resistance of a coil of aluminium wire at 18°C is 200 Ω. The temperature of the wire is increased and the resistance rises to 240 Ω. If the temperature coefficient of resistance of aluminium is 0.0039/°C at 18°C determine the temperature to which the coil has risen.

Let the temperature rise to θ°

Resistance at θ°C, $R_\theta = R_{18}[1 + \alpha_{18}(\theta - 18)]$

i.e. $240 = 200[1 + (0.0039)(\theta - 18)]$

$$240 = 200 + (200)(0.0039)(\theta - 18)$$
$$240 - 200 = 0.78(\theta - 18)$$
$$40 = 0.78(\theta - 18)$$
$$\frac{40}{0.78} = \theta - 18$$
$$51.28 = \theta - 18, \text{ from which, } \theta = 51.28 + 18 = 69.28°\text{C}$$

Hence the temperature of the coil increases to 69.28°C

If the resistance at 0°C is not known, but is known at some other temperature θ_1, then the resistance at any temperature can be found as follows:

$$R_1 = R_0(1 + \alpha_0\theta_1) \text{ and } R_2 = R_0(1 + \alpha_0\theta_2)$$